

# MAT 1341 C Partial Exam I

March 4, 2019      Length: 80 minutes.

Professor: Rachid Bentoumi

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Student number: \_\_\_\_\_

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2	
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## PLEASE CAREFULLY READ THESE INSTRUCTIONS:

1. You have 80 minutes to write this exam.
2. You are not allowed to consult your notes or any books. Calculators, phones, and other electronic devices are not allowed.
3. Carefully read each question and **answer all questions in the space provided for this purpose.** For questions 4 to 6, you can use the back of the pages if necessary, but do not forget to indicate it to the T.A!
4. Questions 1 to 3 are multiple choice questions, each worth 1 point. No partial credit will be awarded. You must indicate the method you used to select the correct answer; unjustified answers will not be given credit.
5. Questions 4 to 6 are to be developed. They are each worth 6 points. **You must justify and write your answers correctly to get all possible points.**
6. Good luck! Bonne chance!

1. For a homogeneous system of 2019 equations with 269 unknowns, which of the following statements is true?

- A. The system may be inconsistent
- B. The system is never inconsistent and never has a unique solution
- C. The system is never inconsistent and always has a unique solution
- D. The system is never inconsistent and can have a unique solution
- E. The system is never inconsistent and has a solution with 1750 parameters
- F. The system has at least 1750 solutions

2. Which of the following sets are bases of  $\mathbb{R}^4$ ?

(1)  $\{(1, 0, 0, 0), (4, 2, 0, 0), (1, 4, 0, 0), (5, 6, 1, 0)\}$

(2)  $\{(0, -1, 2, 3), (0, 3, 3, 2), (4, 2, 0, 0)\}$

(3)  $\{(-1, 3, -5, 1), (1, -2, 4, 2), (2, 0, 4, 3), (5, 1, 9, 4), (4, 2, 0, 0)\}$

A. All three are bases.

B. Only (1) is a basis.

C. Only (2) is a basis.

D. (1) and (2) are both bases.

E. (1) and (3) are both bases.

F. None of these three are bases of  $\mathbb{R}^4$

**3.** Find the value of  $s$  for which  $(1, -4, s)$  belongs to  $\text{Span}\{(1, 2, 1), (2, -2, 6), (4, 14, 0)\}$

A.  $s = 0$

B.  $s = 1$

C.  $s = 2$

D.  $s = 3$

E.  $s = 4$

F.  $s = 5$

4. Consider the vector space  $\mathbb{F}([-1, 1]) = \{f \mid f: [-1, 1] \rightarrow \mathbb{R}\}$  of functions, defined from  $[-1, 1]$  to  $\mathbb{R}$ , with the standard operations. Recall that the zero vector of  $\mathbb{F}([-1, 1])$  is the function that has the value 0 for all  $x \in [-1, 1]$ .

Define three functions  $f$ ,  $g$  and  $h$  de  $\mathbb{F}([-1, 1])$  by

$$f(x) = x, \quad g(x) = 1 - 2x + x^2, \quad \text{and} \quad h(x) = 1 + x^3.$$

Now let,  $W = \text{Span} \{f, g, h\}$

- a) Show that  $f$ ,  $g$  and  $h$  are linearly independent.
- b) Find the base of  $W$  and determine the dimension of  $W$ .
- c) If  $k \in \mathbb{F}([-1, 1])$  is the function defined by  $k(x) = 2 + x^2 + x^3$ , for  $x \in [-1, 1]$ , show that  $k \in W$ .
- d) What is the dimension of the subspace  $Y = \text{Span} \{f, g, h, k, \}$ ?  
(You must justify your answers)



5. Consider the following linear system:

$$\begin{array}{ccccccc} x & + & y & + & 2z & = & 1 \\ 2x & + & y & + & az & = & 2 \\ 3x & + & 4y & + & az & = & c \end{array}$$

- a) Calculate the rank of the coefficient matrix  $A$  and the rank of the augmented matrix  $[A|b]$ , in function of the parameters  $a$  and  $c$ .
- b) Deduce the values of  $a$  and  $c$  for which the system has
- (i) no solutions
  - (ii) infinitely many solutions
  - (iii) unique solution.
- c) In case b (ii) above, find the general solution of the system and give a complete geometrical description of the set of solutions.





6. In each case, indicate in the corresponding box if the statement below is (always) true or can be false.

- If you think that the statement may be wrong, give an explicit example that it is false.
- If you think that the statement is (always) true, you must justify it with a clear explanation.

I. A system of 10 linear equations with 15 unknowns is consistent with many infinitely solutions.

ANSWER

II. If the coefficient matrix  $A$  of a homogeneous linear system of 8 equations with 7 unknowns such that  $\text{rank}(A) = 7$ , then the system has a unique solution.

ANSWER

- III. If the coefficient matrix of a consistent linear system has a column of zeros, then the system has many infinitely solutions.

ANSWER

- IV. Consider two vectors  $u$  and  $v$  from a vector space  $V$ .

a) Give the definition of " $\{u, v\}$  is a linearly independent set."

b) If  $\{u, v\}$  is a base of  $\mathbb{R}^2$ , then  $\{u - v, v\}$  is linearly independent.

ANSWER